STEM Innovation Academy of Orange
NJIT's MTH 111: Calculus I
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Subject: Calculus Unit Title: Function, Limits and Continuity Grade: 12	Teacher: Ahmed Salama Duration: 3 weeks

Summary of Unit

The idea of limits is essential in Calculus because it helps us develop definitions, formulas and theorems. This unit will set us up for the study of derivatives.
During this unit, students will....
- compute various limits (regular limits, one-sided limits, limits as you approach infinity....)
- be able to determine limits graphically, algebraically, numerically, and through tables
- be able to apply limits to explain the behavior of functions near a given point.
- be able to use limits to understand the concept of continuity.
- use limits to talk about the asymptotic and unbounded behaviors of a function
- use limits to talk about the types of discontinuity in given functions

Unit Overview

Unit 1: Functions, Limits and continuity
Essential Questions
➢ How do limits serve as a means to better understand functions and their behavior? ➢ Why are limits essential to the concept of continuity? ➢ Why are limits essential to the underlying meaning of calculus? ➢ How can the concept of a limit be used as a analogy to real-world situations?
Enduring Understandings
➢ limits are useful in describing real life situations ➢ limits can be solved in using a various methods ➢ the concept of limit can be extended to include one-sided limits, limits to infinity, and infinite limits ➢ limits may not exist for some functions at a given value of x ➢ numerical and graphical information can be used to estimate limits ➢ limits of various functions (limits of sums, differences, products, quotients, etc.) can be found using ➢ basic theorems of limits and algebraic rules ➢ limit of a function can be found using algebraic manipulation, alternate forms of trig functions, and ➢ squeeze (sandwich theorem) ➢ asymptotic and unbounded behavior of functions can be explained and described using limits ➢ limits are used to define continuity ➢ there are different types of discontinuity (jump, removable, infinite discontinuity) ➢ continuity is an essential condition for theorems such as Intermediate Value Theorem, Extreme Value ➢ Theorem, and Mean Value Theorem

Common Core State Standards
➢ Identify vertical asymptotes in rational and logarithmic functions by identifying locations where the function value approaches infinity; estimate limits numerically and graphically; calculate limits analytically: • algebraic simplification • direct substitution • one-sided limits • rationalization ➢ LC.1.C.2 Calculate infinite limits and use the result to identity vertical asymptotes in rational and logarithmic functions ➢ LC.1.C.3 Calculate limits at infinity and use the result to identify horizontal asymptotes in rational and exponential functions ➢ LC.1.C.4 Calculate limits at infinity and use the result to identify unbounded behavior in rational, exponential, and logarithmic functions ➢ LC.1.C.5 ➢ Identify and classify graphically, algebraically, and numerically if a discontinuity is removable or non-removable; identify the three conditions that must exist in order for a function to be continuous at $x=a$ • $f(a)$ is defined • the limit as x approaches a of $f(x)$ $=f(a)$ • the limit as approaches a of exists ➢ LC.1.C.6 Apply the Intermediate Value Theorem for continuous functions

Scope and Sequence

Overview		
Lesson	Topic	Suggesting Pacing and Dates
1	Functions and Their Graphs	2-3 days
2	Find limits of functions graphically and numerically	4-5 days
3	Evaluating Limits Analytically	4-5days
4	Continuity and One – Sided Limits	4-5 days
5	Review and Exam	1-2 days

Assessment Framework

Assessment	CCSS	Estimated Time	Format
Diagnostic/Readiness Assessment (Beginning of Unit)	A.CED.2, F.IF.5, LC1.LC2	½ Block	Individual
Assessment Check Up 1 (After lesson 5)	F.IF.7.c, LC1.C.3	½ Block	Individual
Performance (Critical Area) Task Graphing from Roots	A.APR2, LC3.LC4	½ Block	Individual
Check up 2 Ron Larson 5 quiz *p 115*	L.C.1C2.L.C2.C3.L1.C3	½ Block	Individual
Unit 1 Assessment	F.IF.7.C,L.C.1C2.L.C2.C3.L1.C3	1 Block	Individual
Performance (Modeling)Task Introduction to Limits College Fund	A.REI.11,F.IF.7.c,L.C.1C2.L.C2.C3.L1.C3	1 Block	Pair/group
Others			

Lesson Analysis

Lesson 1: Functions and Their Graphs

Objective

- Use function notation to represent and evaluate a function.
- Find the domain and range of a function.
- Sketch the graph of a function.
- Classify combinations of functions and transformations.

Focused Mathematical Practices

- MP2: Reason abstractly and quantitatively
- MP 5: Use appropriate tools strategically
- MP 7: Look for and make use of structure

Vocabulary: Function notation, vertical transformation, horizontal transformation, domain, range

Perquisite Question/s:

How do we find domain and range of more than one equation for one function?
How do you write the function after mixed transformations?

Common Misconceptions:

- Classify the function and find domain and range.
- Graph the function and transform it.

Most relevant CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
F.IF.7.c: Graph l functions, identifying zeros when suitable factorizations are available and showing end behavior	***Review*** • Vocabulary (degree, standard form, monomial, polynomial, cubic, radical, multistep functions. leading term, increasing / decreasing, end-behavior) ***New*** • If the function is Polynomial it can be classified by degree or number of terms • A function is increasing when the y-values increase as x-values increase, and is decreasing when y-values decrease as x-values increase	***Review*** • Create a table for a pattern / write a pattern rule • Identify a function`s type and plot important points to graph. • Write a function in standard form in order to more easily classify by degree ***New*** • X-intercept/s, y-intercept/s • Asymptotes and concavity.	online textbook resources Video/ khan Academy and You-Tube	2-3 days	Lesson Check: pg. 43 (use as exit slip)

Lesson 2: Find limits of functions graphically and numerically

Objective

- Estimate a limit using a numerical or a graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of a limit.

Focused Mathematical Practices

- MP 1: Make sense of problems and persevere in solving them
- MP 2: Use appropriate tools strategically
- MP 3: Construct viable arguments and critique the reasoning of others

Vocabulary: Factor theorem, multiple zero, linear factor, multiplicity, relative maximum, relative minimum

Common Misconceptions:

- Errors with determining limits of step function.
- Confuse between vertical and horizontal asymptotes.
- Rationalize before finding the limits could be confusion point.

Perquisite Question/s:

- How do limits serve as a means to better understand functions and their behavior?
- Why are limits essential to the concept of continuity?
- Why are limits essential to the underlying meaning of calculus?

Most Relevant CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
L.C.1C2: Find the limits of functions algebraically and graphically **A.APR.3:** Identify zeros/ behavior of the function to find the limits.	***Review*** • Students will know the differences between the limits of function and its value. New • $lim_{x\to 0}\frac{!x!}{x}$ *does not ex* • Unbounded behavior • Vertical and Horizontal asymptotes.	***Review*** • students need to be skilled at algebraically manipulating functions • students need to be skilled at reading functions graphically and through tables ***New*** • students need to be skilled at use graphing calculators to determine limits	Online Text Book and Khan Academy	3-4 days	Lesson check: pg. 72

Lesson 3: Evaluating Limits Analytically

Objective

- Evaluating a limit using properties of limits
- Develop and use a strategy for finding limits.
- Evaluate a limit using dividing out and rationalizing techniques.
- Evaluate a limit using the squeeze Theorem.

Focused Mathematical Practices

- MP 1: Make sense of problems and persevere in solving them
- MP 3: Construct viable arguments and critique the reasoning of others.
- MP 5: Use appropriate tools
- MP 6: Attend to precision

Vocabulary: Rationalizing, end behavior model, right handed limits left handed limits, horizontal asymptote, vertical asymptote, oblique asymptote, indeterminate

Common Misconceptions:

- Confuse with limits of rational functions.
- Errors when you find limits of radical functions.

Perquisite Question/s

- Why are limits essential to the underlying meaning of calculus?
- How can the concept of a limit be used as an analogy to real-world situations?
- What could be the limits of sin x and $lim_{x\to 0}\frac{\sin x}{x}$

Most relevant CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
L.C.1, L.C2., L.C3, L.C.4: **Find limits of trigonometric functions, Rationalize function to find the limit.** $\boldsymbol{lim_{x\to 0}\frac{\sqrt{x+1}-2}{x-3}}$ **A.SSE.2** Use the structure of a trig-function to simplify limits example: $lim_{x\to 0}\frac{Cos\,x-1}{x}$	***Review*** • Limits with radical functions ***New*** • Limits involve trigonometric functions	***Review*** • ***Rationalize technique.*** ***New*** • Determine limits of trigonometric functions.	• Factoring • Eliminating • Rationalizing • Trig functions value of Zero and 90^0 • Trig functions relationships • EBook, Edu-Media	3-4days	Lesson Check pg. 83

Lesson 4: Continuity and One – Sided Limits

Objective

- Determine continuity at a point and an open interval.
- Determine one side limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the intermediate theorem.

Focused Mathematical Practices

- MP 3: Construct viable arguments and critique the reasoning of others.
- MP 7: Look for and make use of structure
- MP 6: Attend to precision

Vocabulary: Testing Continuity, Cusp, jump over limits, spider limit, drum limit, destroy continuity.

Common Misconceptions:

- Existing limits but function is not continuous.
- Left side limits different than right side.

Perquisite question/s

- How to investigate continuity of function?
- How to discuss continuity of a graphed function?

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.APR.2: Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x) **A.APR.6:** Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a	***Review:*** * clearly present methods, reasoning, justifications, and conclusions ***New*** • Analyze functions for intervals of continuity or points of discontinuity (Level of Difficulty - 3 • Analysis)	***Review*** • Find limits of a function with multiple steps. ***New*** • Identify types of discontinuity (removable, jump, infinite, etc.) (Level of Difficulty - 2 • Comprehension) • ▸develop conjectures based on exploration with technology; Existing of function.	Lesson 1-4 online test-book Divide using algebra tiles: http://www.doe.virginia.gov/testing/solsearch/sol/math/A/m_ess_a-2b_1.pdf	3-4 days	Lesson Check pg. 94 Class discussion #8 challenge

computer algebra system.					

Lesson 5: Infinite Limits

Objective

- Determine infinite limits from the left and from the right.
- Find and Sketch the vertical and horizontal asymptotes of a function.

Focused Mathematical Practices

- MP 1: Make sense of problems and persevere in solving them
- MP 4: Model with mathematics
- MP 5: Use appropriate tools strategically

Vocabulary: Vertical asymptote, horizontal asymptote, rational functions with infinite limits.

Common Misconceptions:

- Undefined limits and vertical asymptotes.
- Choosing a model that doesn't *best* fit a data set

Perquisite/Question/s

- Determine infinite limit and limit
- In your own expression write a function with vertical asymptote x=-2, x= 6 and with a zero at x=3

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
L.C.1, L2.C5, L4.C5. Find the limits of rational function for infinite, methods of finding limits of rational function that end with zero, finding limits of rational function that ends with negative or positive infinite. Applications	***Review*** ▸ identify how mathematical characteristics of functions and its limit are related in different representations; ***New*** • limits are needed for developing various theorems (Intermediate Value Theorem, Extreme Value Theorem, and Mean Value Theorem)	***Review*** • ***Find Limit of rational functions.*** ***New*** • Use the method of dividing by the greatest power term to find the limit of functions. • Use calculator to estimate the limits of different types of functions	EBook, Online resources College Board Exam sample questions.	3-4 day	Lesson check pg. 107 1 – 6 (5 & 6 challenge) Inter college test Chapter 7